NAME: ________________________ STUDENT #___________

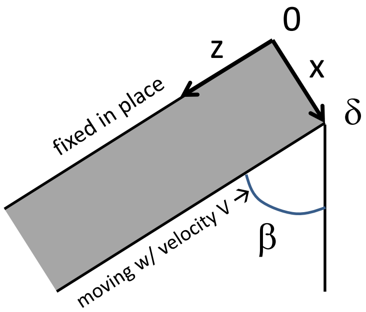
TN4780ta and AESB2320, 2014-15
Part 1 Final Examination - 11 March

This exam can count toward the Part 1 score for either TN4780ta or AESB2320.
Circle here which course you wish the exam to count toward:

AESB2320 TN4780ta

Turn in this exam with your answer sheet.
Write your solutions *on your answer sheet*, not here. In all cases ***show your work***.
To avoid any possible confusion,
state the equation numbers and figure numbers of equations and figures you use.
Beware of unnecessary information in the problem statement.

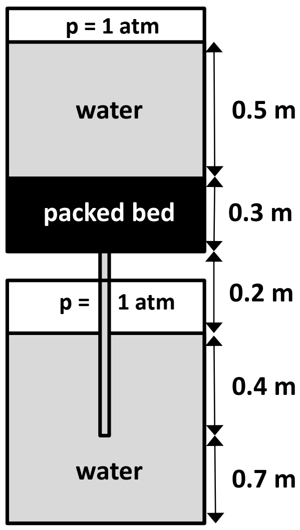
1. Newtonian fluid (density ρ, viscosity μ) is contained between two parallel flat plates a distance δ apart. The plates are held at an angle β to the vertical as shown at right. The top plate is fixed in place, while the bottom plate moves upward (in the negative-z direction) with velocity $V < 0$.



   a. Derive an equation for the velocity of the fluid as a function of position in the gap between the plates, $v_z(x)$. (25 pts)
   b. Under what conditions does *all* the fluid flow upwards: i.e., under what conditions is $v_z(x) \leq 0$ for the entire gap? Be quantitative in your answer. (Only 5 pts - don't spend too long on this if you don't get it.)

   Note: the derivation of the falling film from BSL Sect. 2.2 (1^{st} edition) is given at the back of this exam.
   (30 points)

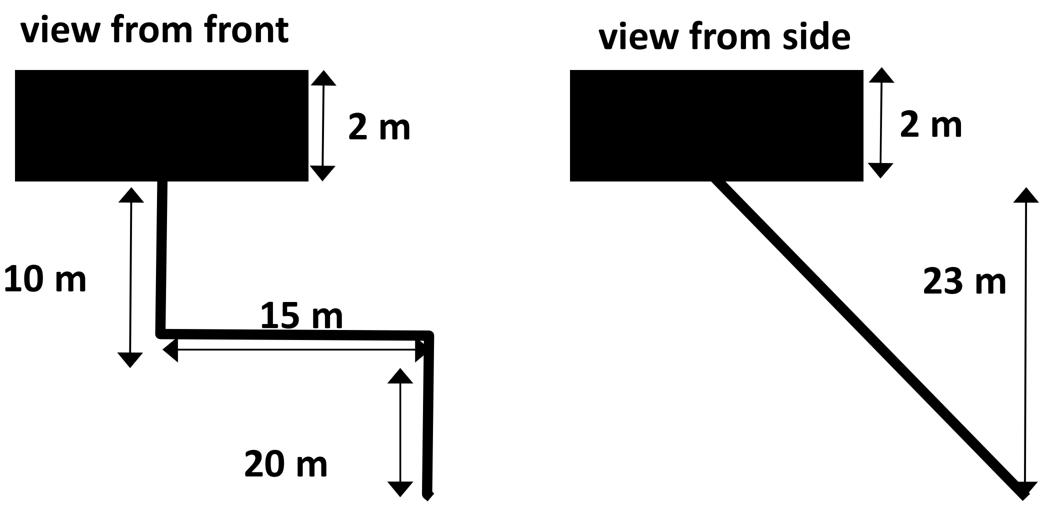
2. Water (density 1000 kg/m^3, viscosity 0.001 Pa s) flows through a packed bed (a water purifier) in a setup shown at right. The packing contains particles 5 mm in diameter and has porosity 0.4. There is a layer of stagnant water above the packed bed, and the discharge of the filter is below the level of water in the bottom tank, as shown. Above the water in both tanks, the pressure is 1 atm. The packed bed is cylindrical, 0.3 m thick and 0.1 m in diameter. Assume all the flow potential is dissipated in the packed bed, not in any of the other tubes or fittings of the apparatus. (Note picture not drawn to scale.)



   a. What is the potential gradient driving the flow of water through the packed bed?
   b. What is the total flow rate of water Q through the packed bed, in m^3/s?
   (25 points)

3. A drainage system is designed to carry rainwater off the top of a hill to a river below. The pipe is 0.2 m in diameter, and its "height of protuberances" 0.0008 m. The properties of water are given in problem 2. There are two sharp (not rounded) 90° elbows and in the pipe plus a sharp (not rounded) constriction at the entrance, as shown below. The pipe discharges into the air above the river. The pipe comprises 10, 15 and 20-m segments (going down the slope, sideways, and down the slope), but because it is set on the side of the hill the change in elevation along the pipe is only 23 m. At the top of the pipe is a reservoir of water of depth 2 m. An engineer wants to know the water velocity through the pipe.
   a. Write out the equation that must be satisfied by the velocity in the pipe. Plug in all the numbers you can into this equation.
   b. Solve this equation for velocity in the pipe. Start by assuming that the flow is highly turbulent (very large Re).

   (35 points)



4. Rocky proposes that one could eliminate the need for locks in canals by dissolving material in the water in the canal to turn it into a Bingham plastic. Suppose the canal were 1 m deep, with side walls very far (infinitely far) apart. The bottom surface of the canal, of course, does not move. Suppose the density of the fluid is 1000 kg/m^3. Suppose the canal tilts at an angle 89.8° to the vertical (i.e., it is close to horizontal). What yield stress τ_o would be required to prevent flow of the Bingham plastic downward through the canal?

   (10 points)

<u>properties of water</u>

$\mu = 0.001$ Pa s $\quad\quad \rho = 1000$ kg/m^3

rate of z-momentum out across surface at $x + \Delta x$	$(LW)(\tau_{xz})\|_{x+\Delta x}$	(2.2–2)
rate of z-momentum in across surface at $z = 0$	$(W \Delta x\, v_z)(\rho v_z)\|_{z=0}$	(2.2–3)
rate of z-momentum out across surface at $z = L$	$(W \Delta x\, v_z)(\rho v_z)\|_{z=L}$	(2.2–4)
gravity force acting on fluid	$(LW \Delta x)(\rho g \cos \beta)$	(2.2–5)

Note that we always take the "in" and "out" directions in the direction of the positive x- and z-axes (in this problem these happen to coincide with the direction of momentum transport). The notation $|_{x+\Delta x}$ means "evaluated at $x + \Delta x$."

When these terms are substituted into the momentum balance of Eq. 2.1–1, we get

$$LW\tau_{xz}|_x - LW\tau_{xz}|_{x+\Delta x} + W \Delta x\, \rho v_z^2|_{z=0} - W \Delta x\, \rho v_z^2|_{z=L} + LW \Delta x\, \rho g \cos \beta = 0 \quad (2.2\text{–}6)$$

Because v_z is the same at $z = 0$ as it is at $z = L$ for each value of x, the third and fourth terms just cancel one another. We now divide Eq. 2.2–6 by $LW\, \Delta x$ and take the limit as Δx approaches zero:

$$\lim_{\Delta x \to 0} \left(\frac{\tau_{xz}|_{x+\Delta x} - \tau_{xz}|_x}{\Delta x} \right) = \rho g \cos \beta \quad (2.2\text{–}7)$$

The quantity on the left side may be recognized as the definition of the first derivative of τ_{xz} with respect to x. Therefore, Eq. 2.2–7 may be rewritten as

$$\frac{d}{dx} \tau_{xz} = \rho g \cos \beta \quad (2.2\text{–}8)$$

This is the differential equation for the momentum flux τ_{xz}. It may be integrated to give

$$\tau_{xz} = \rho g x \cos \beta + C_1 \quad (2.2\text{–}9)$$

The constant of integration may be evaluated by making use of the boundary condition at the liquid-gas interface (see §2.1):

B.C. 1: at $x = 0$, $\tau_{xz} = 0$ (2.2–10)

Substitution of this boundary condition into Eq. 2.2–9 reveals that $C_1 = 0$. Hence the momentum-flux distribution is

$$\boxed{\tau_{xz} = \rho g x \cos \beta} \tag{2.2–11}$$

as shown in Fig. 2.2–2.

If the fluid is Newtonian, then we know that the momentum flux is related to the velocity gradient according to

$$\tau_{xz} = -\mu \frac{dv_z}{dx} \tag{2.2–12}$$

Substitution of this expression for τ_{xz} into Eq. 2.2–11 gives the following differential equation for the velocity distribution:

$$\frac{dv_z}{dx} = -\left(\frac{\rho g \cos \beta}{\mu}\right) x \tag{2.2–13}$$

This equation is easily integrated to give

$$v_z = -\left(\frac{\rho g \cos \beta}{2\mu}\right) x^2 + C_2 \tag{2.2–14}$$

The constant of integration is evaluated by using the boundary condition

B.C. 2: $\quad$ at $x = \delta, \quad v_z = 0$ $\quad$ (2.2–15)

Substitution of this boundary condition into Eq. 2.2–14 shows that $C_2 = (\rho g \cos \beta/2\mu)\delta^2$. Therefore, the velocity distribution is

$$\boxed{v_z = \frac{\rho g \, \delta^2 \cos \beta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]} \tag{2.2–16}$$

Hence the velocity profile is parabolic. (See Fig. 2.2–2.)

Once the velocity profile has been found, a number of quantities may be calculated:

(i) The *maximum velocity* $v_{z,\max}$ is clearly the velocity at $x = 0$; that is

$$v_{z,\max} = \frac{\rho g \, \delta^2 \cos \beta}{2\mu} \tag{2.2–17}$$